Sequences and Series

Assertion Reason Questions

Direction: In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).
Choose the correct answer out of the following choices.
(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(c) (A) is true but (R) is false.
(d) (A) is false but (R) is true.

1. Assertion (A): If 5^{th} and 8^{th} term of a G.P be 48 and 384 respectively, then the common ratio of G.P is 2.
Reason (R): If 18, x, 14 are in A.P, then x = 16.

Ans. (b) Both (A) and (R) are true but (R) is not the correct explanation of (A).
Explanation: We have,

$$T_5 = 48$$

We know that

$$\therefore \quad ar^4 = 48 \quad \text{...(i)}$$

$$T_8 = 384$$

$$T_n = ar^{n-1}$$

$$ar^7 = 384 \quad \text{...(ii)}$$

So, on dividing eq. (ii) by (i), we get

$$\frac{ar^7}{ar^4} = \frac{384}{48}$$

$$r^3 = 8$$

$$r = 2$$

Given, 18, n, 14 are in $A.P$
So,

$$\text{Common ratio} = n - 18$$

$$= 14 - n$$

$$2n = 32$$

$$n = 16$$

2. Assertion (A): The sum of the first 20 terms of an A.P, 4, 8, 12 is equal to 840.

Reason (R): Sum of n terms of an A.P is

$\left[S_n = \frac{n}{2}(2a + (n-1)d\right]$.

Ans. (a) Both (A) and (R) are true and (R) is the correct explanation of (A).

Explanation: Given A.P. is 4, 8, 12

a = 4 and d = 4

We know that

$$S_n = \frac{n}{2} 2a + (n-1)d$$

Now, $S_{20} = \frac{20}{2}\{2 \times 4 + (20-1) \times 4\}$

$S_{20} = 10(8 + 76)$

$= 10 \times 84$

$= 840$

3. Assertion (A): The sum of the series $\frac{3}{\sqrt{5}} + \frac{4}{\sqrt{5}} + \sqrt{5} + \dots 25$ term is $75\sqrt{5}$.

Reason (R): If 27, n, 3 are in G.P then $x = \pm 4$.

Ans. (c) (A) is true but (R) is false.

Explanation:

Given, $S_n = \frac{3}{\sqrt{5}} + \frac{4}{\sqrt{5}} + \sqrt{5} + \dots$

Here, $a = \frac{3}{\sqrt{5}}$ and $d = \frac{1}{\sqrt{5}}$

So, $S_n = \frac{25}{2}\left[2 \times \frac{3}{\sqrt{5}} + (25-1)\frac{1}{\sqrt{5}}\right]$

$= 25 \times \frac{15}{5} \times \sqrt{5} = 75\sqrt{5}$

Given, 27, n, 3 are in G.P.

$$\frac{n}{27} = \frac{3}{n}$$

So, $n^2 = 81$

$\Rightarrow$ $n = \pm 9$

4.

Assertion (A): If the number $\frac{-2}{7}, k, \frac{-7}{2}$ is in G.P then $k = \pm 1$.

Reason (R): If a_1, a_2, a_3 are in G.P then $\frac{a_2}{a_1} = \frac{a_3}{a_2}$.

Ans. (a) Both (A) and (R) are true and (R) is the correct explanation of (A).

Explanation: We know that, if a_1 a_2 a_3 are in

G.P. then $\frac{a_2}{a_1} = \frac{a_3}{a_2}$

If $\frac{-2}{7}, k, \frac{-7}{2}$ are in G.P.

Then, $\frac{a_2}{a_1} = \frac{a_3}{a_2}$

$$\frac{k}{-\frac{2}{7}} = \frac{-\frac{7}{2}}{k}$$

$$14k^2 = 14$$

$$k^2 = 1$$

$$k = \pm 1$$

5. Assertion (A): The sum of first n terms of the series 0.6+0.66 +0.666 + ... is

$$\frac{2}{3}\left[n - \frac{1}{9}\left\{1 - \left(\frac{1}{10}\right)^n\right\}\right].$$

Reason (R): General term of a GP is

$T_n = ar^{n-1}$, where a = first term

and r = common ratio.

Ans. (b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

Explanation: Let

S = 0.6+0.66+0.666+ .... upto n terms

S = 6(0.1 0.11 +0.111+ .... upto n terms)

$$= \frac{6}{9}(0.9 + 0.99 + 0.999 + \ldots \text{ upto } n \text{ terms})$$

$$= \frac{2}{3}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \ldots \text{upto } n \text{ terms}\right]$$

$$= \frac{2}{3}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + \left(1 - \frac{1}{1000}\right) + \ldots \text{ upto } n \text{ terms}\right]$$

$$= \frac{2}{3}\left[(1 + 1 + 1 + \ldots \text{ upto } n \text{ terms}) - \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \ldots \text{upto } n \text{ terms}\right)\right]$$

$$= \frac{2}{3}\left[n - \frac{\frac{1}{10}\left\{1 - \left(\frac{1}{10}\right)^n\right\}}{1 - \frac{1}{10}}\right]$$

$$\left[\because \text{ sum of } GP = \frac{a(1 - r^n)}{1 - r}, r < 1\right]$$

$$= \frac{2}{3}\left[n - \frac{\frac{1}{10}\left\{1 - \left(\frac{1}{10}\right)^n\right\}}{\frac{9}{10}}\right]$$

$$= \frac{2}{3}\left[n - \frac{1}{9}\left\{1 - \left(\frac{1}{10}\right)^n\right\}\right]$$

6. Assertion (A): The sum of first n natural number is 1+ 2+ 3+ ... + n

$$= n\left(\frac{n+1}{2}\right).$$

Reason (R): For n number of A.P. the sum

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right].$$

Ans. (a) Both (A) and (R) are true and (R) is the correct explanation of (A).
Explanation: We know that the sum of first n

natural number is $n\left(\frac{n+1}{2}\right)$

For A.P.

$1 + 2 + \dots n$

$a = 1, d = 1$

$$S_n = \frac{n}{2}\left[2\times1+(n-1)1\right]$$

$$= \frac{n}{2}[2+n-1]$$

$$= \frac{n}{2}[n+1]$$

Hence, both assertion and reason are true and reason is the correct explanation of assertion.

7. Assertion (A): Two sequence can be in both A.P. and G.P.
Reason (R): If the sum of n terms of
a sequence is a quadratic
expression, then it always
represents an A.P.

Ans. (c) (A) is true but (R) is false.
Explanation: Two sequence can be in both A.P. and G.P. in some special cases. If the sum of n term of a sequence is quadratic then it may or may not be represent an A.P.